Lining behaviour -
Analytical solutions of coupled segmented rings in soil

- analytical formulation
- interpretation
- comparing with measurements

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This document is part of the Preliminary Thesis “Design Philosophy of Concrete Linings of Shield Driven Tunnels in soft soils”, Chapters 3, 4 and 5
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3.4 Ovalisering en Buigende Moment
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  3.5.2 Reducerende stijfheid
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<th>Page</th>
</tr>
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</tr>
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</table>
5 Comparing Full Scale Tests with Analytical solutions

In [2] results are presented from a full scale test on a three rings lining. This chapter provides verification of the formulated analytical solutions of the coupled system of rings with the results of the full scale tests.

5.1 Geometry

The test setup has three rings with varying positions of the longitudinal joints placed in vertical position. The middle ring (called ring 1) has the first longitudinal joint at position $i=0.5$. The top and bottom ring (called ring 2) have the first longitudinal joint at position $i=0$.

Comparing results will be based on the next data:

- system radius $r_s$
- thickness of segments $d$
- Modulus of Elasticity of concrete $E_b$
- Modulus of Elasticity of ground $E_g$
- Contact length longitudinal joint $b$
- Analysed width of ring $s_2$
- ovalisation loading $s_0$
- Compression loading $k_v$
- Coupling stiffness

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s$</td>
<td>4525mm</td>
</tr>
<tr>
<td>$d$</td>
<td>400mm</td>
</tr>
<tr>
<td>$E_b$</td>
<td>40000Mpa</td>
</tr>
<tr>
<td>$E_g$</td>
<td>0 (no ground, only loading)</td>
</tr>
<tr>
<td>$b$</td>
<td>170mm</td>
</tr>
<tr>
<td>$s_2$</td>
<td>750mm</td>
</tr>
<tr>
<td>$s_0$</td>
<td>0.0185Mpa</td>
</tr>
<tr>
<td>$k_v$</td>
<td>0.4295Mpa</td>
</tr>
<tr>
<td></td>
<td>$1*10^5N/mm$</td>
</tr>
</tbody>
</table>

5.2 Loading at once

5.2.1 Deformations due to ovalisation

The first step is to calculate a system of coupled rings with the analytical approach (chapter 3.10).
Matrix (127) makes use of $c_{ri}$ for each longitudinal. In this step all $c_{ri}$ are equal and calculated according to equation (28):

$$c_{ri} = \frac{b l^2 E_b}{12} = \frac{750 \times 170^2 \times 40000}{12} = 7.225 \times 10^{10} \text{Nmm/ rad}$$

Furthermore:

$$B' = k_r r = 1 \times 10^5 \times 4525 = 4.525 \times 10^5 \text{N}$$

Filling in the matrix results in the system stiffness matrix $[S]$: 

$$[S] = \begin{bmatrix}
-3.55 & -2.83 & -1.11 & -0.13 \\
2.01 & -1.00 & 0.00 & 0.00 \\
2.60 & 2.21 & -1.00 & 0.00 \\
2.69 & 3.97 & 2.21 & -1.00
\end{bmatrix}$$

The loading vector (129) also makes use of $c_{ri}$ which are already determined. Next $A'$ is:

$$A' = \sigma_{br} r^2 = 0.0185 \times 750 \times 4525^2 = 2.84 \times 10^5 \text{Nmm}$$

The loading vector is:

$$\{f\} = \begin{bmatrix}
-0.68 \\
1.27 \\
0.67 \\
-0.63
\end{bmatrix}$$

The solution of the system is the displacement difference of the rings: the vector $\{\Delta u\}$:

$$\{\Delta u\} = \begin{bmatrix}
0.48 \\
-0.32 \\
-0.13 \\
0.35
\end{bmatrix} \text{mm}$$

From $\{\Delta u\}$ the following results can be extracted:

<table>
<thead>
<tr>
<th>i</th>
<th>$\Delta u$[mm]</th>
<th>$F_k=K_k\Delta u$[kN]</th>
<th>$\theta_i$[rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.48</td>
<td>48</td>
<td>7.60E-04</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.32</td>
<td>-32</td>
<td>4.67E-04</td>
</tr>
<tr>
<td>1</td>
<td>-0.13</td>
<td>-13</td>
<td>-1.43E-04</td>
</tr>
<tr>
<td>1.5</td>
<td>0.35</td>
<td>35</td>
<td>-7.84E-04</td>
</tr>
<tr>
<td>2</td>
<td>-0.35</td>
<td>-35</td>
<td>-7.84E-04</td>
</tr>
<tr>
<td>2.5</td>
<td>0.13</td>
<td>13</td>
<td>-1.43E-04</td>
</tr>
<tr>
<td>3</td>
<td>0.32</td>
<td>32</td>
<td>4.67E-04</td>
</tr>
<tr>
<td>3.5</td>
<td>-0.48</td>
<td>-48</td>
<td>7.60E-04</td>
</tr>
</tbody>
</table>

Now the rotations in the longitudinal joints are known, the top and side deformations of the rings can be estimated:

- **ring 1 (middle ring)**

Based on the theory in 3.7.2 it can be found that the displacement of point B in Figuur 22:

$$u_{xB} = r \sum_{i=\frac{\pi}{2}}^{\frac{\pi}{2}} \theta_i (1 - \sin \beta_i)$$
Solving this:

<table>
<thead>
<tr>
<th>i</th>
<th>β</th>
<th>θi</th>
<th>θ(1-sinβ)rs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5 (1.5)</td>
<td>282.9</td>
<td>-7.84E-04</td>
<td>-7.00</td>
</tr>
<tr>
<td>6.5 (0.5)</td>
<td>334.3</td>
<td>4.67E-04</td>
<td>3.03</td>
</tr>
<tr>
<td>0.5</td>
<td>25.7</td>
<td>4.67E-04</td>
<td>1.20</td>
</tr>
<tr>
<td>1.5</td>
<td>77.1</td>
<td>-7.84E-04</td>
<td>-0.09</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From uyB rotation $\varphi_A$ is calculated:

$$\varphi_A = \frac{u_{yB}}{2r_s} = \frac{-2.87}{2 \times 4525} = -3.17 \times 10^{-4} \text{ rad}$$

The absolute top displacement is:

$$u_{top} = -\varphi_A r_s - r_s \sum_{n} \theta_i \sin \beta_i$$

Solving this:

<table>
<thead>
<tr>
<th>i</th>
<th>β</th>
<th>θi</th>
<th>θ(1-sinβ)rs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5 (1.5)</td>
<td>282.9</td>
<td>-7.84E-04</td>
<td>3.46</td>
</tr>
<tr>
<td>6.5 (0.5)</td>
<td>334.3</td>
<td>4.67E-04</td>
<td>-0.92</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>2.54</td>
</tr>
</tbody>
</table>

$$u_{top} = 3.17 \times 10^{-4} \times 4525 - 2.54 = -1.11 \text{mm}$$

For the side of the ring the same approach can be used. Based on Figuur 23 for uxB can be formulated:

$$u_{xB} = r_s \sum_{n} \theta_i (1 + \cos \beta_i)$$

<table>
<thead>
<tr>
<th>i</th>
<th>β</th>
<th>θi</th>
<th>θ(1+cosβ)rs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>25.7</td>
<td>4.67E-04</td>
<td>4.01</td>
</tr>
<tr>
<td>1.5</td>
<td>77.1</td>
<td>-7.84E-04</td>
<td>4.34</td>
</tr>
<tr>
<td>2.5</td>
<td>128.6</td>
<td>-1.43E-04</td>
<td>-0.24</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>u_{xB} = -0.57 \text{mm}</td>
</tr>
</tbody>
</table>

$$\varphi_A = \frac{-0.57}{2 \times 4525} = -6.3 \times 10^{-5} \text{ rad}$$

The absolute side displacement is:

$$u_{side} = -\varphi_A r_s + r_s \sum_{n} \theta_i \cos \beta_i$$

<table>
<thead>
<tr>
<th>i</th>
<th>β</th>
<th>θi</th>
<th>θ(cosβ)rs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>25.7</td>
<td>4.67E-04</td>
<td>1.90</td>
</tr>
<tr>
<td>1.5</td>
<td>77.1</td>
<td>-7.84E-04</td>
<td>-0.79</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>u_{side} = 1.11 \text{mm}</td>
</tr>
</tbody>
</table>

$$u_{side} = 6.3 \times 10^{-5} \times 4525 + 1.11 = 1.40 \text{mm}$$

At this point all rotations in the longitudinal joints and the top and side displacements of ring 1 are known. Based on chapter 3.7.1 the field of displacements can be calculated. Because the rings and loading are symmetric on the vertical axis, only the area 0 - 180° will be
considered. It is known that due to ovalisation loading deformations are addition of deformations due to rotation in longitudinal joints and deformations due to bending stiffness.

The deformations due to rotations can be calculated as:

\[ u_{LV,\varphi} = r_i \sum_{\beta_i \geq 0} \theta_i \sin(\varphi - \beta_i) - \cos \varphi u_{y0} - \sin \varphi u_{x0} \]

\[ u_{y0} = -u_{top} = 1.11 \text{mm} \]

\[ u_{x0} = u_{(\varphi=90;uy0=0;ux0=0)} - u_{side} = 1.11 - 1.40 = -0.29 \text{mm} \]

To estimate \( u_{\varphi=90;uy0=0;ux0=0} \) the equation \( u_{LV,\varphi} \) is primary calculated with \( u_{y0}=0 \) and \( u_{x0}=0 \) (second column, Table 1). Next \( u_{\varphi=90;uy0=0;ux0=0} \) is corrected with \( u_{y0} \) and \( u_{x0} \) (third column, Table 1), which results in the total field of deformations due to rotations in longitudinal joints.

Deformation due to bending can be calculated with equation (14):

\[ u_{EI,\varphi} = \frac{4}{3} \frac{r^4}{E_d d^3} \sigma_2 \cos(2\varphi) \]

The total deformations follows from:

\[ u_{tot,\varphi} = u_{LV,\varphi} + u_{EI,\varphi} \]

Solving this equations will look like Table 1:

<table>
<thead>
<tr>
<th>( \varphi ) rad</th>
<th>( u_{LV,\varphi} ) mm</th>
<th>( u_{x0} ) mm</th>
<th>( u_{EI,\varphi} ) mm</th>
<th>( u_{tot,\varphi} ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>-1.11</td>
<td>-4.04</td>
<td>-5.15</td>
</tr>
<tr>
<td>12.9</td>
<td>0.00</td>
<td>-1.02</td>
<td>-3.64</td>
<td>-4.66</td>
</tr>
<tr>
<td>25.7</td>
<td>0.00</td>
<td>-0.87</td>
<td>-2.52</td>
<td>-3.39</td>
</tr>
<tr>
<td>38.6</td>
<td>0.47</td>
<td>-0.22</td>
<td>-0.90</td>
<td>-1.12</td>
</tr>
<tr>
<td>51.4</td>
<td>0.92</td>
<td>0.45</td>
<td>0.90</td>
<td>1.35</td>
</tr>
<tr>
<td>64.3</td>
<td>1.32</td>
<td>1.09</td>
<td>2.52</td>
<td>3.61</td>
</tr>
<tr>
<td>77.1</td>
<td>1.65</td>
<td>1.68</td>
<td>3.64</td>
<td>5.32</td>
</tr>
<tr>
<td>90.0</td>
<td>1.11</td>
<td>1.40</td>
<td>4.04</td>
<td>5.44</td>
</tr>
<tr>
<td>102.9</td>
<td>0.52</td>
<td>1.04</td>
<td>3.64</td>
<td>4.68</td>
</tr>
<tr>
<td>115.7</td>
<td>-0.10</td>
<td>0.63</td>
<td>2.52</td>
<td>3.15</td>
</tr>
<tr>
<td>128.6</td>
<td>-0.71</td>
<td>0.20</td>
<td>0.90</td>
<td>1.10</td>
</tr>
<tr>
<td>141.4</td>
<td>-1.44</td>
<td>-0.40</td>
<td>-0.90</td>
<td>-1.29</td>
</tr>
<tr>
<td>154.3</td>
<td>-2.09</td>
<td>-0.97</td>
<td>-2.52</td>
<td>-3.49</td>
</tr>
<tr>
<td>167.1</td>
<td>-2.63</td>
<td>-1.49</td>
<td>-3.64</td>
<td>-5.13</td>
</tr>
<tr>
<td>180.0</td>
<td>-3.05</td>
<td>-1.94</td>
<td>-4.04</td>
<td>-5.98</td>
</tr>
</tbody>
</table>

Table 1 Solving for deformations

The results from Table 1 are graphicly presented in Figuur 54, together with measurements from the full scale testing and also a calculation with the frame work program. It is pretty clear that results are very close.
ring 2 (bottom and top ring)

Based on the theory in 3.7.2 it can be found that the displacement of point B in Figuur 22:

\[
u_{yB} = r_i \sum_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \theta_i (1 - \sin \beta_i)
\]

Solving this:

<table>
<thead>
<tr>
<th>(i)</th>
<th>(\theta)</th>
<th>(\theta_i)</th>
<th>(\theta(1-\sin\beta_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (1)</td>
<td>308.6</td>
<td>-1.43E-04</td>
<td>-1.15</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>7.60E-04</td>
<td>3.44</td>
</tr>
<tr>
<td>1</td>
<td>51.4</td>
<td>-1.43E-04</td>
<td>-0.14</td>
</tr>
<tr>
<td>Sum</td>
<td>u_{yB}= 2.14mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From \(u_{yB}\) rotation \(\varphi_A\) is calculated:

\[
\varphi_A = \frac{u_{yB}}{2r_s} = \frac{2.14}{2*4525} = 2.37*10^{-4} rad
\]

The absolute top displacement is:

\[
u_{top} = -\varphi_A r_s - r_i \sum_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \theta_i \sin \beta_i
\]

Solving this:

<table>
<thead>
<tr>
<th>(i)</th>
<th>(\beta)</th>
<th>(\theta_i)</th>
<th>(\theta(\sin\beta_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>308.6</td>
<td>-1.43E-04</td>
<td>0.51</td>
</tr>
<tr>
<td>Sum</td>
<td>u_{top}= -1.58mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the side of the ring the same approach can be used. Based on Figuur 23 for \(u_{xB}\) can be formulated:
\[ u_{xB} = r_s \sum_0^\pi \theta_i (1 + \cos \beta_i) \]

<table>
<thead>
<tr>
<th>i</th>
<th>( \beta )</th>
<th>( \theta_i )</th>
<th>( \theta(1+\cos\beta)r_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>7.60E-04</td>
<td>6.88</td>
</tr>
<tr>
<td>1</td>
<td>51.4</td>
<td>-1.43E-04</td>
<td>-1.05</td>
</tr>
<tr>
<td>2</td>
<td>102.9</td>
<td>-7.84E-04</td>
<td>-2.76</td>
</tr>
<tr>
<td>3</td>
<td>154.3</td>
<td>4.67E-04</td>
<td>0.21</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>3.04mm</td>
</tr>
</tbody>
</table>

\[ \varphi_A = \frac{3.28}{2*4525} = 3.63*10^{-4} \text{ rad} \]

The absolute side displacement is:

\[ u_{side} = -\varphi_A r_s + r_s \sum_0^\pi \theta_i \cos \beta_i \]

<table>
<thead>
<tr>
<th>i</th>
<th>( \beta )</th>
<th>( \theta_i )</th>
<th>( \theta(\cos\beta)r_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>7.60E-04</td>
<td>3.44</td>
</tr>
<tr>
<td>1</td>
<td>51.4</td>
<td>-1.43E-04</td>
<td>-0.40</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>3.04mm</td>
</tr>
</tbody>
</table>

\[ u_{side} = 3.63*10^{-4} * 4525 + 3.04 = 1.39 \text{ mm} \]

At this point all rotations in the longitudinal joints and the top and side displacements of ring 2 are known. Based on chapter 3.7.1 The field of displacements can be calculated. Because the rings and loading are symmetric on the vertical axis, only the area 0 - 180° will be considered. It is known that due to ovalisation loading deformations are addition of deformations due to rotation in longitudinal joints and deformations due to bending stiffness. The deformations due to rotations can be calculated as:

\[ u_{L,\varphi} = r_s \sum_{\beta_i < \varphi} \theta_i \sin(\varphi - \beta_i) - \cos \varphi u_{y0} - \sin \varphi u_{x0} \]

\[ u_{y0} = -u_{x0} = 1.58 \text{ mm} \]

\[ u_{x0} = u_{y0,xy=0;xz=0} - u_{side} = 3.04 - 1.39 = 1.65 \text{ mm} \]

To estimate \( u_{\varphi=90;yz=0;xz=0} \) the equation \( u_{L,\varphi} \) is primary calculated with \( u_{y0}=0 \) and \( u_{x0}=0 \) (second column, Table 1). Next \( u_{\varphi=90;yz=0;xz=0} \) is corrected with \( u_{y0} \) and \( u_{x0} \) (third column, Table 1), which results in the total field of deformations due to rotations in longitudinal joints.

Deformation due to bending can be calculated with equation (14):

\[ u_{EL,\varphi} = \frac{4}{3} \frac{r^4}{E_s d^3} \sigma_2 \cos(2\varphi) \]

The total deformations follows from:

\[ u_{tot,\varphi} = u_{L,\varphi} + u_{EL,\varphi} \]

Solving this equations will look like Table 2:
<table>
<thead>
<tr>
<th>$\varphi$ [rad]</th>
<th>$u_{y,0}$ [mm]</th>
<th>$u_{y,0} + u_{x,0}$ [mm]</th>
<th>$u_{EI,0}$ [mm]</th>
<th>$u_{tot,0}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
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<td>-4.78</td>
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<td>0.90</td>
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<td>3.64</td>
<td>4.76</td>
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<td>1.40</td>
<td>4.04</td>
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</tr>
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<td>3.64</td>
<td>5.24</td>
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<td>1.73</td>
<td>0.93</td>
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</tr>
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<td>0.90</td>
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<td>-1.40</td>
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<td>-1.20</td>
<td>-2.52</td>
<td>-3.72</td>
</tr>
<tr>
<td>167.1</td>
<td>-2.54</td>
<td>-1.37</td>
<td>-3.64</td>
<td>-5.01</td>
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<td>180.0</td>
<td>-3.05</td>
<td>-1.47</td>
<td>-4.04</td>
<td>-5.51</td>
</tr>
</tbody>
</table>

Table 2 Solving for deformations

The results from Table 2 are graphicly presented in Figuur 55, together with measurements from the full scale testing. Again the analytical results are very close to measurements.

### Comparing Deformations

Comparing Deformations

Full Scale Tests / Analytical Solution

![Comparing Deformations](image_url)

Figuur 55: Comparing Deformations ring 2

#### 5.2.2 Longitudinal joints

In chapter 5.2 the analytical approach has been given to draw a comparison for deformations between the analytical solution and measured data of the full scale test. Comparing the deformations give satisfying parallel between the analytical solution and measured data. It has to be verified that the analytical solution has been based on longitudinal joint rotation stiffness constants $c_{ri}$ that agree with theoretical values (chapter 3.5). In the analytical
solution $c_i=7.225 \times 10^{10} \text{Nm/rad}$ for all longitudinal joints. From the analytical solution rotations have been derived:

$$\beta \Delta \theta_i \quad F_i = k_i \Delta u_i \quad \theta_i \quad c_i \quad M_i$$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\beta$</th>
<th>$\Delta u_i \text{[mm]}$</th>
<th>$F_i = k_i \Delta u_i \text{[kN]}$</th>
<th>$\theta_i \text{[rad]}$</th>
<th>$c_i$</th>
<th>$M_i \text{[kNm]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.48</td>
<td>48</td>
<td>7.60E-04</td>
<td>7.225E+10</td>
<td>55</td>
</tr>
<tr>
<td>0.5</td>
<td>25.7</td>
<td>-0.32</td>
<td>-32</td>
<td>4.67E-04</td>
<td>7.225E+10</td>
<td>34</td>
</tr>
<tr>
<td>1</td>
<td>51.4</td>
<td>-0.13</td>
<td>-13</td>
<td>-1.43E-04</td>
<td>7.225E+10</td>
<td>-10</td>
</tr>
<tr>
<td>1.5</td>
<td>77.1</td>
<td>0.35</td>
<td>35</td>
<td>-7.84E-04</td>
<td>7.225E+10</td>
<td>-57</td>
</tr>
<tr>
<td>2</td>
<td>102.9</td>
<td>-0.35</td>
<td>-35</td>
<td>-7.84E-04</td>
<td>7.225E+10</td>
<td>-57</td>
</tr>
<tr>
<td>2.5</td>
<td>128.6</td>
<td>0.13</td>
<td>13</td>
<td>-1.43E-04</td>
<td>7.225E+10</td>
<td>-10</td>
</tr>
<tr>
<td>3</td>
<td>154.3</td>
<td>0.32</td>
<td>32</td>
<td>4.67E-04</td>
<td>7.225E+10</td>
<td>34</td>
</tr>
<tr>
<td>3.5</td>
<td>180.0</td>
<td>-0.48</td>
<td>-48</td>
<td>7.60E-04</td>
<td>7.225E+10</td>
<td>55</td>
</tr>
</tbody>
</table>

Table 3: Results from analytical solution

The turning point of linear rotation stiffness to reduced rotation stiffness is defined in equation (31):

$$M \geq \frac{Nl_p}{6}$$

For $N$ is calculated (1)

$$N = \sigma_0 \cdot r \cdot b = 0.4295 \times 4525 \times 750 = 1457 \text{kN}$$

The turning point is:

$$M \geq \frac{Nl_p}{6} \geq \frac{1457 \times 10^3 \times 170}{6} = 41 \text{kNm}$$

It has to be concluded that at $i=0, 1.5, 2$ and $3.5$ the assumed rotation stiffness has been too stiff. As a consequence the analytical solution has to be recalculated with adapted rotation stiffness constants. Based on Table 3 new values for rotation stiffness can be estimated and the solutions are recalculated. Again rotation stiffness will be verified resulting in a smaller discrepancy in rotation stiffness. Here an iterative process involves the calculation and finally rotation stiffness constants are used which agree the theoretical values.

Results after a few iterations are presented in

$$\beta \Delta \theta_i \quad F_i = k_i \Delta u_i \quad \theta_i \quad c_i \quad M_i$$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\beta$</th>
<th>$\Delta u_i \text{[mm]}$</th>
<th>$F_i = k_i \Delta u_i \text{[kN]}$</th>
<th>$\theta_i \text{[rad]}$</th>
<th>$c_i$</th>
<th>$M_i \text{[kNm]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.50</td>
<td>50</td>
<td>7.76E-04</td>
<td>6.819E+10</td>
<td>53</td>
</tr>
<tr>
<td>0.5</td>
<td>25.7</td>
<td>-0.31</td>
<td>-31</td>
<td>4.74E-04</td>
<td>7.225E+10</td>
<td>34</td>
</tr>
<tr>
<td>1</td>
<td>51.4</td>
<td>-0.13</td>
<td>-13</td>
<td>-1.44E-04</td>
<td>7.225E+10</td>
<td>-10</td>
</tr>
<tr>
<td>1.5</td>
<td>77.1</td>
<td>0.35</td>
<td>35</td>
<td>-8.62E-04</td>
<td>6.578E+10</td>
<td>-57</td>
</tr>
<tr>
<td>2</td>
<td>102.9</td>
<td>-0.35</td>
<td>-35</td>
<td>-8.62E-04</td>
<td>6.578E+10</td>
<td>-57</td>
</tr>
<tr>
<td>2.5</td>
<td>128.6</td>
<td>0.13</td>
<td>13</td>
<td>-1.44E-04</td>
<td>7.225E+10</td>
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<td>3</td>
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<tr>
<td>3.5</td>
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<td>-50</td>
<td>7.76E-04</td>
<td>6.819E+10</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 4: Results from analytical solution after iteration

Comparing the results from Table 3 and Table 4 show that in this case the iteration was not quite influencing the results. Rotation stiffness constants only decreased 9% or 5% in several longitudinal joints. Influence on bending moments in the longitudinal joints is minimal. For completeness the two graphs for displacements are given again, with adapted rotation stiffness values.
Comparing Deformations
Full Scale Tests / Analytical Solution / Frame work

Figuur 56: Comparing Deformations ring 1, adapted rotation stiffnesses

Comparing Deformations
Full Scale Tests / Analytical Solution

Figuur 57: Comparing Deformations ring 2, adapted rotation stiffnesses

M-phi Relation longitudinal joints
joint rotations in full scale test

Figuur 58: Comparing M-phi relation anlyical solution and theory, adapted rotation stiffnesses
5.2.3 Tangential Stresses in Segments

From the analytical solution bending moments are known at all positions with longitudinal joints (Table 5).

<table>
<thead>
<tr>
<th>i</th>
<th>β</th>
<th>$F_i$ [kN]</th>
<th>M ring 1 [kNm]</th>
<th>M ring 2 [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>48</td>
<td>53</td>
<td>136</td>
</tr>
<tr>
<td>0.5</td>
<td>25.7</td>
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<td>85</td>
<td>33</td>
</tr>
<tr>
<td>1</td>
<td>51.4</td>
<td>-13</td>
<td>-10</td>
<td>-32</td>
</tr>
<tr>
<td>1.5</td>
<td>77.1</td>
<td>35</td>
<td>-114</td>
<td>-57</td>
</tr>
<tr>
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<td>102.9</td>
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<td>3.5</td>
<td>180.0</td>
<td>-48</td>
<td>136</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 5: Bending Moments, results from analytical solution

Within the known bending moments from Table 5 bending moments can be calculated by interpolation. As a result the total field of bending moments can be determined for ring 1 and 2. Tangential stresses due to bending moments can easily be calculated from:

$$\sigma_M = \frac{6M}{bh^3}$$

From the full scale tests stresses known because strains have been measured. Next a comparison will be made between analytically calculated stresses and stresses known from the full scale testing. Besides results from FEM calculations will be presented [3].
Observing Figuur 59 and Figuur 60 the results of the analytical solution suit well with both FEM solution and measurements. It is mentioned that comparison only took place for values at the centre lines of segments (i.e. not near ring joints) and that stress values near longitudinal joints are not as easy to estimated as supposed. Near the joints measurements show a high disturbance due to side effects of concrete. These effects are also known from FEM solutions. This disturbances will be discussed later.

### 5.3 Sequential loading